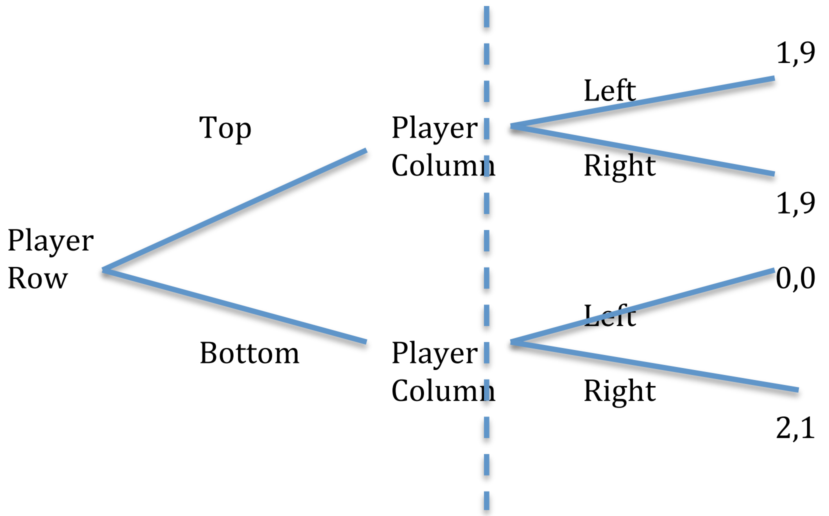
Remember the example:




We found the following strategies for the players, in response to their beliefs:

Player Row forms beliefs about Player Column's actions. $\pi_L$ represents Player Row's belief of Player Column's probability of choosing Left. Therefore, $1 - \pi_L$ represents Player Row's belief that Player Column plays Right.

Player Row plays Top with probability Pt and plays Bottom with probability Pb=1-Pt.

Hence, for Player Row:

$$\begin{cases} Pt = 1 & if & \pi_L > \frac{1}{2} & - CASE\ 1 \\ Pt \in [0,1] & if & \pi_L = \frac{1}{2} & - CASE\ 2 \\ Pt = 0 & if & \pi_L < \frac{1}{2} & - CASE\ 3 \end{cases}$$

Player Column also forms beliefs about Player Row's actions. $\pi_T$ represents Player Column's belief of Player Row's probability of choosing Top. Therefore, $1 - \pi_T$ represents Player Column's belief that Player Row plays Bottom.

Player Column plays Left with probability $P_L$ and plays Right with probability $P_R$=1-$P_L$.

Hence, for Player Column:

$$\begin{cases} P_L = 1 & if & \pi_T > 1 & IMPOSSIBLE \\ P_L \in [0,1] & if & \pi_T = 1 & \\ P_L = 0 & if & \pi_T < 1 & \end{cases}$$

To find the equilibria we must find consistency in the beliefs a player has, his chosen action in response to that belief, and also that the other player (guessing that response) would actually choose an action in the first place that is consistente with the initial belief of the first player.

We can analyse this by looking at the three possible scenarios for Player Row:

**CASE 1:**

Player Row believes that $\pi_L > \frac{1}{2}$
In this case, Player Row would choose $Pt = 1$

Note: This can be an equilibrium if Player Columns "guesses" that Pt=1 (i.e., Player Column believes that $\pi_T = 1$ and in reply chooses $P_L$ consistent with Player Row's initial assumption $\pi_L > \frac{1}{2}$ .

Let's check. If Player Column guesses $\pi_T = 1$, Player Column can choose $P_L \in [0,1]$.

Is this consistent with Player Row's initial belief that $\pi_L > \frac{1}{2}$ ? Yes, it can be as long as $P_L \in \left(\frac{1}{2}, 1\right]$.

**We have an equilibrium in which $\pi_L = P_L \in \left(\frac{1}{2}, 1\right]$ and $\pi_T = Pt = 1$.**

**CASE 2:**

Player Row believes that $\pi_L = \frac{1}{2}$
In this case, Player Row could choose anything, i.e., $Pt \in [0,1]$.

We must now check in Player Column's strategies what he can do in reply to having beliefs of $\pi_T \in [0,1]$. We have to check two possible sub-cases: $\pi_T = 1$ and $\pi_T < 1$.

**SUB-CASE 2.1**: Player Column believes that $\pi_T = 1$.

In this case, Player Column is indifferent and can play anything: $P_L \in [0,1]$

Is this consistent with the initial belief that Player Row had that $\pi_L = \frac{1}{2}$ ?
Yes, it is, because ½ is included in the interval [0,1]

**So we have another equilibrium in which $\pi_L = P_L = \frac{1}{2}$ and $\pi_T = Pt = 1$.**
If you note carefully, this equilibrium of case 2.1 together with the one found in CASE 1 "complete" each other.
We can say that: $\pi_L = P_L \in \left[\frac{1}{2}, 1\right]$ and $\pi_T = Pt = 1$

**SUB-CASE 2.2:** Player Column believes that $\pi_T < 1$

In this case Player Column would reply with action $P_L = 0$.
This is inconsistent with the initial assumption of CASE 2, that $\pi_L = \frac{1}{2}$.
There is no equilibrium here.

**CASE 3:** Player Row believes that $\pi_L < \frac{1}{2}$

In this case Player Row would choose $Pt = 0$

If Player Column guessed that $\pi_T = 0$, he would choose $P_L = 0$ .
This is consistent with the initial assumption of case 3 that $\pi_L < \frac{1}{2}$

**So, we found another equilibrium in which $\pi_L = P_L = 0$ and $\pi_T = Pt = 0$.**

NOTE: We could also, more simply, visualize the equilibria in a graph, by considering explicitly in the strategies of the two players that equilibria requires $\pi_L = P_L$ and $\pi_t = P_t$.

Again:

Player Row:

$$\begin{cases} Pt = 1 & if & P_L > \frac{1}{2} \\ Pt \in [0,1] & if & P_L = \frac{1}{2} \\ Pt = 0 & if & P_L < \frac{1}{2} \end{cases}$$

Player Column:

$$\begin{cases} P_L = 1 & if & P_T > 1 \quad IMPOSSIBLE \\ P_L \in [0,1] & if & P_T = 1 \\ P_L = 0 & if & P_T < 1 \end{cases}$$

In a graph we can represent both players and find equilibria when they cross: